Supplementary Material:

Predicting a Set of Minimal Free Energy RNA Secondary Structures Common to Two Sequences

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Dynamic Programming Algorithm Recursions:

V(i,j,k,l) is the lowest free energy sum of a substructure in sequence one with i and j paired and a structure in sequence two with k paired to l with i aligned to k and j aligned to l. W(i,j,k,l) is the lowest free energy sum of a nucleotide fragment from nucleotides i to k in sequence one and k to l in sequence two, such that the fragments will appear in multibranch loops, i is aligned to k, and j is aligned to l.

For calculations done with chemical modification constraints, a third $O(N^2M^2)$ array is utilized, called VMOD(i,j,k,l). VMOD(i,j,k,l) = V(i,j,k,l) except for cases in which i,j,k, or l is a modified nucleotide as explained below.

$V(i,j,k,l) = \min[V_{hairpin}, V_{internal/stack}, V_{multibranch} + Penalty(i,j) + Penalty(k,l)]$ for $j \leq N_1$ and $l \leq N_2$.

$V(i,j,k,l) = \min[V_{hairpin}, V_{internal/stack}, V_{multibranch} + Penalty(i,j) + Penalty(k,l), V_{exterior} + Penalty(i,j) + Penalty(k,l)]$ for $j > N_1$ and $l > N_2$.

where Penalty(a,b) is the A-U/G-U helix end penalty that is applied only to terminal A-U or G-U pairs (Xia *et al.*, 1998).

If either of those base pairs is non-canonical or forbidden by a constraint, V(i,j,k,l) is given large positive free energy. $V_{hairpin}$ considers hairpin loops closed by base pairs i-j and k-l:

$$V_{hairpin} = \Delta G^\circ_{hairpin}(i,j) + \Delta G^\circ_{hairpin}(k,l) + (\Delta G^\circ_{gap})|j-i-l+k|$$

where $\Delta G^\circ_{hairpin}(a,b)$ is the free energy for a hairpin closed by a base pair between nucleotides a and b (Mathews *et al.*, 2004).

$V_{internal/stack}$ is the lowest sum of free energies for a helix extension, bulge loop, or internal loop in the common structure and requires a search through parameters i', j', k', and l' so that:

$V_{internal/stack} = \min[V(i',j',k',l') + \Delta G^\circ_{motif\ 1} + \Delta G^\circ_{motif\ 2}]$ for all $i < i' < j' < j$ and $k < k' < l' < l$, $i'-M \leq k' \leq i'+M$, $j'-M \leq l' \leq j'+M$.

To restrict the overall algorithm to $O(N^3M^3)$, the depth of the search is limited to $i < i' \leq i+M'$, $j-M' \leq j' < j$, $k < k' \leq k + M'$, $l-M' \leq l' < l$. Currently, M' = 20.

$\Delta G^\circ_{motif\ 1}$ is the free energy of the motif closed by pairs i-j and i'-j' and $\Delta G^\circ_{motif\ 2}$ is similarly the energy for the motif in sequence two (Mathews *et al.*, 2004; Mathews *et al.*, 1999; Xia *et al.*, 1998). For example, in sequence one, if i' = i+1 and j'=j-1, this is a continuation of a canonical helix. For loops, if either i' = i+1 or j'=j-1, then this motif is a bulge loop, but is otherwise an internal loop.

For calculations done with chemical modification constraints, VMOD(i',j',k',l') is substituted for V(i',j',k',l') for cases in which either one or both of the motifs is the continuation of a canonical helix.

VMOD(i,j,k,l) is the minimum of the same terms as V(i,j,k,l) except when i,j,k, or l is accessible to chemical modification. If i or j is modified, the i-j base pair is not a G-U pair, and the i'-j' base pair is not a G-U pair, then the terms that account for the canonical continuation of a helix in sequence one are not included in the minimization. Similar cases for modified k and l involving sequence two are also excluded in the determination of VMOD(i,j,k,l).

$V_{multibranch}$ is the lowest free energy sum for multibranch loops closed by pairs i-j and k-l and requires a search through parameters i' and k', with i < i' < j and k < k' < l.

$$V_{multibranch} = \min[V_{multibranch\text{-}1}, V_{multibranch\text{-}2}, V_{multibranch\text{-}3}, V_{multibranch\text{-}4}, V_{multibranch\text{-}5}, V_{multibranch\text{-}6}, V_{multibranch\text{-}7}, V_{multibranch\text{-}8}, V_{multibranch\text{-}9}, V_{multibranch\text{-}10}, V_{multibranch\text{-}11}, V_{multibranch\text{-}12}, V_{multibranch\text{-}13}, V_{multibranch\text{-}14}, V_{multibranch\text{-}15}. V_{multibranch\text{-}16}]$$

$$V_{multibranch\text{-}1} = W(i+1,i',k+1,k') + W(i'+1,j-1,k'+1,l-1) + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop}$$

$$V_{multibranch\text{-}2} = W(i+1,i',k+1,k') + W(i'+1,j-1,k'+1,l-2) + \Delta G^\circ_{danglel\ l-1} + \Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}3} = W(i+1,i',k+2,k') + W(i'+1,j-1,k'+1,l-1) + \Delta G^\circ_{dangle\ k+1} + \Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}4} = W(i+1,i',k+2,k') + W(i'+1,j-1,k'+1,l-2) + \Delta G^\circ_{dangle\ k+1} + \Delta G^\circ_{danglel\ l-1} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}5} = W(i+1,i',k+1,k') + W(i'+1,j-2,k'+1,l-1) + \Delta G^\circ_{dangle\ j-1} + \Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}6} = W(i+1,i',k+1,k') + W(i'+1,j-2,k'+1,l-2) + \Delta G^\circ_{dangle\ j-1} + \Delta G^\circ_{danglel\ l-1} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop}$$

$$V_{multibranch\text{-}7} = W(i+1,i',k+2,k') + W(i'+1,j-2,k'+1,l-1) + \Delta G^\circ_{dangle\ j-1} + \Delta G^\circ_{dangle\ k+1} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}8} = W(i+1,i',k+2,k') + W(i'+1,j-2,k'+1,l-2) + \Delta G^\circ_{dangle\ j-1} + \Delta G^\circ_{dangle\ k+1} + \Delta G^\circ_{danglel\ l-1} + 3\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}9} = W(i+2,i',k+1,k') + W(i'+1,j-1,k'+1,l-1) + \Delta G^\circ_{dangle\ i+1} + \Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}10} = W(i+2,i',k+1,k') + W(i'+1,j-1,k'+1,l-2) + \Delta G^\circ_{dangle\ i+1} + \Delta G^\circ_{danglel\ l-1} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}11} = W(i+2,i',k+2,k') + W(i'+1,j-1,k'+1,l-1) + \Delta G^\circ_{dangle\ i+1} + \Delta G^\circ_{dangle\ k+1} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop}$$

$$V_{multibranch\text{-}12} = W(i+2,i',k+2,k') + W(i'+1,j-1,k'+1,l-2) + \Delta G^\circ_{dangle\ i+1} + \Delta G^\circ_{dangle\ k+1} + \Delta G^\circ_{danglel\ l-1} + 3\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}13} = W(i+2,i',k+1,k') + W(i'+1,j-2,k'+1,l-1) + \Delta G^\circ_{dangle\ i+1} + \Delta G^\circ_{dangle\ j-1} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{gap}$$

$$V_{multibranch\text{-}14} = W(i+2,i',k+1,k') + W(i'+1,j-2,k'+1,l-2) + \Delta G^\circ_{dangle\ i+1} + \Delta G^\circ_{dangle\ j-1} + \Delta G^\circ_{danglel\ l-1} + 3\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{MBL\ closure} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{gap}$$

$V_{multibranch\text{-}15} = W(i+2,i',k+2,k') + W(i'+1,j-2,k'+1,l-1) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ j-1} + \Delta G°_{dangle\ k+1} + 3\Delta G°_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G°_{MBL\ closure} + 2\Delta G°_{helix\ terminating\ in\ MBL\ loop} + \Delta G°_{gap}$

$V_{multibranch\text{-}16} = W(i+2,i',k+2,k') + W(i'+1,j-2,k'+1,l-2) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ j-1} + \Delta G°_{dangle\ k+1} + \Delta G°_{danglel\ l-1} + 4\Delta G°_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G°_{MBL\ closure} + 2\Delta G°_{helix\ terminating\ in\ MBL\ loop}$

$V_{exterior}$ is the lowest free energy sum of exterior loops (loops that contain the ends of the sequence) for sequences one and two.

$V_{exterior} = \min[V_{exterior\text{-}1}, V_{exterior\text{-}2}, V_{exterior\text{-}3}, V_{exterior\text{-}4}, V_{exterior\text{-}5}, V_{exterior\text{-}6}, V_{exterior\text{-}7}, V_{exterior\text{-}8}, V_{exterior\text{-}9}, V_{exterior\text{-}10}, V_{exterior\text{-}11}, V_{exterior\text{-}12}, V_{exterior\text{-}13}, V_{exterior\text{-}14}, V_{exterior\text{-}15}, V_{exterior\text{-}16}]$

$V_{exterior\text{-}1} = W5(j-1-N_1,l-1-N_2) + W3(i+1,k+1)$

$V_{exterior\text{-}2} = W5(j-1-N_1,l-2-N_2) + W3(i+1,k+1) + \Delta G°_{dangle\ l-1} + \Delta G°_{gap}$

$V_{exterior\text{-}3} = W5(j-1-N_1,l-1-N_2) + W3(i+1,k+2) + \Delta G°_{dangle\ k+1} + \Delta G°_{gap}$

$V_{exterior\text{-}4} = W5(j-1-N_1,l-2-N_2) + W3(i+1,k+2) + \Delta G°_{dangle\ k+1} + \Delta G°_{dangle\ l-1} + 2\Delta G°_{gap}$

$V_{exterior\text{-}5} = W5(j-2-N_1,l-1-N_2) + W3(i+1,k+1) + \Delta G°_{dangle\ j-1} + \Delta G°_{gap}$

$V_{exterior\text{-}6} = W5(j-2-N_1,l-2-N_2) + W3(i+1,k+1) + \Delta G°_{dangle\ j-1} + \Delta G°_{dangle\ l-1}$

$V_{exterior\text{-}7} = W5(j-2-N_1,l-1-N_2) + W3(i+1,k+2) + \Delta G°_{dangle\ k+1} + \Delta G°_{dangle\ j-1} + 2\Delta G°_{gap}$

$V_{exterior\text{-}8} = W5(j-2-N_1,l-2-N_2) + W3(i+1,k+2) + \Delta G°_{dangle\ k+1} + \Delta G°_{dangle\ j-1} + \Delta G°_{dangle\ l-1} + \Delta G°_{gap}$

$V_{exterior\text{-}9} = W5(j-1-N_1,l-1-N_2) + W3(i+2,k+1) + \Delta G°_{dangle\ i+1} + \Delta G°_{gap}$

$V_{exterior\text{-}10} = W5(j-1-N_1,l-2-N_2) + W3(i+2,k+1) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ l-1} + 2\Delta G°_{gap}$

$V_{exterior\text{-}11} = W5(j-1-N_1,l-1-N_2) + W3(i+2,k+2) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ k+1}$

$V_{exterior\text{-}12} = W5(j-1-N_1,l-2-N_2) + W3(i+2,k+2) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ k+1} + \Delta G°_{dangle\ l-1} + \Delta G°_{gap}$

$V_{exterior\text{-}13} = W5(j-2-N_1,l-1-N_2) + W3(i+2,k+1) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ j-1} + 2\Delta G°_{gap}$

$V_{exterior\text{-}14} = W5(j-2-N_1,l-2-N_2) + W3(i+2,k+1) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ j-1} + \Delta G°_{dangle\ l-1} + \Delta G°_{gap}$

$V_{exterior\text{-}15} = W5(j-2-N_1,l-1-N_2) + W3(i+2,k+2) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ k+1} + \Delta G°_{dangle\ j-1} + \Delta G°_{gap}$

$V_{exterior\text{-}16} = W5(j-2-N_1,l-2-N_2) + W3(i+2,k+2) + \Delta G°_{dangle\ i+1} + \Delta G°_{dangle\ k+1} + \Delta G°_{dangle\ j-1} + \Delta G°_{dangle\ l-1}$

W(i,j,k,l) is the minimum of three terms:

$W(i,j,k,l) = \min[W_{W\ extend}, W_{branch}, W_{bifurcation}]$

$W_{W\ extend}$ is a recursive extension of a smaller W fragment and is the minimum of 16 terms:

$$W_{W\ extend} = \min[W_{W\ extend\text{-}1}, W_{W\ extend\text{-}2}, W_{W\ extend\text{-}3}, W_{W\ extend\text{-}4}, W_{W\ extend\text{-}5}, W_{W\ extend\text{-}6}, W_{W\ extend\text{-}7}, W_{W\ extend\text{-}8}, W_{W\ extend\text{-}9}, W_{W\ extend\text{-}10}, W_{W\ extend\text{-}11}, W_{W\ extend\text{-}12}, W_{W\ extend\text{-}13}, W_{W\ extend\text{-}14}, W_{W\ extend\text{-}15}]$$

$$W_{W\ extend\text{-}1} = W(i,j,k,l\text{-}1) + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}2} = W(i,j,k+1,l) + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}3} = W(i,j,k+1,l\text{-}1) + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + 2\Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}4} = W(i,j\text{-}1,k,l) + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}5} = W(i,j\text{-}1,k,l\text{-}1) + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}}$$
$$W_{W\ extend\text{-}6} = W(i,j\text{-}1,k+1,l) + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + 2\Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}7} = W(i,j\text{-}1,k+1,l\text{-}1) + 3\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}8} = W(i+1,j,k,l) + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}9} = W(i+1,j,k,l\text{-}1) + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + 2\Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}10} = W(i+1,j,k+1,l) + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}}$$
$$W_{W\ extend\text{-}11} = W(i+1,j,k+1,l\text{-}1) + 3\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}12} = W(i+1,j\text{-}1,k,l) + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + 2\Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}13} = W(i+1,j\text{-}1,k,l\text{-}1) + 3\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}14} = W(i+1,j\text{-}1,k+1,l) + 3\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}}$$
$$W_{W\ extend\text{-}15} = W(i+1,j\text{-}1,k+1,l\text{-}1) + 4\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}}$$

W_{branch} is the introduction of a helical branch to W(i,j,k,l) and is the minimum of 16 terms that account for the stacking of dangling ends:

$$W_{branch} = \min[W_{branch\text{-}1}, W_{branch\text{-}2}, W_{branch\text{-}3}, W_{branch\text{-}4}, W_{branch\text{-}5}, W_{branch\text{-}6}, W_{branch\text{-}7}, W_{branch\text{-}8}, W_{branch\text{-}9}, W_{branch\text{-}10}, W_{branch\text{-}11}, W_{branch\text{-}12}, W_{branch\text{-}13}, W_{branch\text{-}14}, W_{branch\text{-}15}, W_{branch\text{-}16}]$$

$$W_{branch\text{-}1} = V(i,j,k,l) + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + \text{Penalty}(i,j) + \text{Penalty}(k,l)$$
$$W_{branch\text{-}2} = V(i,j,k,l\text{-}1) + \Delta G^\circ_{\text{dangle } l} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}} + \text{Penalty}(i,j) + \text{Penalty}(k,l\text{-}1)$$
$$W_{branch\text{-}3} = V(i,j,k+1,l) + \Delta G^\circ_{\text{dangle } k} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}} + \text{Penalty}(i,j) + \text{Penalty}(k+1,l)$$
$$W_{branch\text{-}4} = V(i,j,k+1,l\text{-}1) + \Delta G^\circ_{\text{dangle } k} + \Delta G^\circ_{\text{dangle } l} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + 2\Delta G^\circ_{\text{gap}} + \text{Penalty}(i,j) + \text{Penalty}(k+1,l\text{-}1)$$
$$W_{branch\text{-}5} = V(i,j\text{-}1,k,l) + \Delta G^\circ_{\text{dangle } j} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + \Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}} + \text{Penalty}(i,j\text{-}1) + \text{Penalty}(k,l)$$
$$W_{branch\text{-}6} = V(i,j\text{-}1,k,l\text{-}1) + \Delta G^\circ_{\text{dangle } j} + \Delta G^\circ_{\text{dangle } l} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \text{Penalty}(i,j\text{-}1) + \text{Penalty}(k,l\text{-}1)$$
$$W_{branch\text{-}7} = V(i,j\text{-}1,k+1,l) + \Delta G^\circ_{\text{dangle } j} + \Delta G^\circ_{\text{dangle } k} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + 2\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + 2\Delta G^\circ_{\text{gap}} + \text{Penalty}(i,j\text{-}1) + \text{Penalty}(k+1,l)$$
$$W_{branch\text{-}8} = V(i,j\text{-}1,k+1,l\text{-}1) + \Delta G^\circ_{\text{dangle } j} + \Delta G^\circ_{\text{dangle } k} + \Delta G^\circ_{\text{dangle } l} + 2\Delta G^\circ_{\text{helix terminating in MBL loop}} + 3\Delta G^\circ_{\text{unpaired nucleotide in MBL loop}} + \Delta G^\circ_{\text{gap}} + \text{Penalty}(i,j\text{-}1) +$$

$\text{Penalty}(k+1,l-1)$

$W_{branch\text{-}9} = V(i+1,j,k,l) + \Delta G^\circ_{dangle\ i} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + \Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + \Delta G^\circ_{gap} + \text{Penalty}(i+1,j) + \text{Penalty}(k,l)$

$W_{branch\text{-}10} = V(i+1,j,k,l-1) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ l} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{gap} + \text{Penalty}(i+1,j) + \text{Penalty}(k,l-1)$

$W_{branch\text{-}11} = V(i+1,j,k+1,l-1) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ k} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + \text{Penalty}(i+1,j) + \text{Penalty}(k+1,l-1)$

$W_{branch\text{-}12} = V(i+1,j,k+1,l-1) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ k} + \Delta G^\circ_{dangle\ l} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 3\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + \Delta G^\circ_{gap} + \text{Penalty}(i+1,j) + \text{Penalty}(k+1,l-1)$

$W_{branch\text{-}13} = V(i+1,j-1,k,l) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ j} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 2\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + 2\Delta G^\circ_{gap} + \text{Penalty}(i+1,j-1) + \text{Penalty}(k,l)$

$W_{branch\text{-}14} = V(i+1,j-1,k,l-1) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ j} + \Delta G^\circ_{dangle\ l} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 3\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + \Delta G^\circ_{gap} + \text{Penalty}(i+1,j-1) + \text{Penalty}(k,l-1)$

$W_{branch\text{-}15} = V(i+1,j-1,k+1,l) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ j} + \Delta G^\circ_{dangle\ k} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 3\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + \Delta G^\circ_{gap} + \text{Penalty}(i+1,j-1) + \text{Penalty}(k+1,l)$

$W_{branch\text{-}16} = V(i+1,j-1,k+1,l-1) + \Delta G^\circ_{dangle\ i} + \Delta G^\circ_{dangle\ k} + \Delta G^\circ_{dangle\ j} + \Delta G^\circ_{dangle\ l} + 2\Delta G^\circ_{helix\ terminating\ in\ MBL\ loop} + 4\Delta G^\circ_{unpaired\ nucleotide\ in\ MBL\ loop} + \text{Penalty}(i+1,j-1) + \text{Penalty}(k+1,l-1)$

$W_{bifurcation}$ accounts for bifurcations in W(i,j,k,l) so that multibranch loops with more than three branching helices can be predicted:

$W_{bifurcation} = \min[W(i,i',k,k') + W(i'+1,j,k'+1,l)]$ for all $i < i' < j$ and $k < k' < l$, but with $i' - M \le k' \le i' + M$.

W5(i,k) is the minimum of four terms:

$W5(i,k) = \min[W5(i-1,k) + \Delta G^\circ_{gap},\ W5(i,k-1) + \Delta G^\circ_{gap},\ W5(i-1,k-1) + \Delta G^\circ_{gap},\ W5_{bifurcation}]$

Where $W5_{bifurcation}$ is a a minimum of 16 terms and requires a search over i' and k' with $i' < i$ and $k' < k$ and $i' - M \le k \le i' + M$:

$W5_{bifurcation} = \min[W5_{bifurcation\text{-}1}, W5_{bifurcation\text{-}2}, W5_{bifurcation\text{-}3}, W5_{bifurcation\text{-}4}, W5_{bifurcation\text{-}5}, W5_{bifurcation\text{-}6}, W5_{bifurcation\text{-}7}, W5_{bifurcation\text{-}8}, W5_{bifurcation\text{-}9}, W5_{bifurcation\text{-}10}, W5_{bifurcation\text{-}11}, W5_{bifurcation\text{-}12}, W5_{bifurcation\text{-}13}, W5_{bifurcation\text{-}14}, W5_{bifurcation\text{-}15}, W5_{bifurcation\text{-}16}]$

$W5_{bifurcation\text{-}1} = W5(i',k') + V(i'+1,i,k'+1,k) + \text{Penalty}(i'+1,i) + \text{Penalty}(k'+1,k)$

$W5_{bifurcation\text{-}2} = W5(i',k') + V(i'+1,i,k',k-1) + \Delta G^\circ_{dangle\ k} + \Delta G^\circ_{gap} + \text{Penalty}(i'+1,i) + \text{Penalty}(k',k-1)$

$W5_{bifurcation\text{-}3} = W5(i',k') + V(i'+1,i,k'+2,k) + \Delta G^\circ_{dangle\ k'+1} + \Delta G^\circ_{gap} + \text{Penalty}(i'+1,i) + \text{Penalty}(k'+2,k)$

$W5_{\text{bifurcation-4}} = W5(i',k') + V(i'+1,i,k'+2,k-1) + \Delta G^\circ_{\text{dangle }k'+1} + \Delta G^\circ_{\text{dangle }k} + 2\Delta G^\circ_{gap} + \text{Penalty}(i'+1,i) + \text{Penalty}(k'+2,k-1)$

$W5_{\text{bifurcation-5}} = W5(i',k') + V(i'+1,i-1,k',k) + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{gap} + \text{Penalty}(i'+1,i-1) + \text{Penalty}(k',k)$

$W5_{\text{bifurcation-6}} = W5(i',k') + V(i'+1,i-1,k'+1,k-1) + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{\text{dangle }k} + \text{Penalty}(i'+1,i-1) + \text{Penalty}(k'+1,k-1)$

$W5_{\text{bifurcation-7}} = W5(i',k') + V(i'+1,i-1,k'+2,k) + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{\text{dangle }k'+1} + 2\Delta G^\circ_{gap} + \text{Penalty}(i'+1,i-1) + \text{Penalty}(k'+2,k)$

$W5_{\text{bifurcation-8}} = W5(i',k') + V(i'+1,i-1,k'+2,k-1) + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{\text{dangle }k'+1} + \Delta G^\circ_{\text{dangle }k} + \Delta G^\circ_{gap} + \text{Penalty}(i'+1,i-1) + \text{Penalty}(k'+2,k-1)$

$W5_{\text{bifurcation-9}} = W5(i',k') + V(i'+2,i,k'+1,k) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{gap} + \text{Penalty}(i'+2,i) + \text{Penalty}(k'+1,k)$

$W5_{\text{bifurcation-10}} = W5(i',k') + V(i'+2,i,k'+1,k-1) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }k} + 2\Delta G^\circ_{gap} + \text{Penalty}(i'+2,i) + \text{Penalty}(k'+1,k-1)$

$W5_{\text{bifurcation-11}} = W5(i',k') + V(i'+2,i,k'+2,k) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }k'+1} + \text{Penalty}(i'+2,i) + \text{Penalty}(k'+2,k)$

$W5_{\text{bifurcation-12}} = W5(i',k') + V(i'+2,i,k'+2,k-1) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }k'+1} + \Delta G^\circ_{\text{dangle }k} + \Delta G^\circ_{gap} + \text{Penalty}(i'+2,i) + \text{Penalty}(k'+2,k-1)$

$W5_{\text{bifurcation-13}} = W5(i',k') + V(i'+2,i-1,k'+1,k) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }i} + 2\Delta G^\circ_{gap} + \text{Penalty}(i'+2,i-1) + \text{Penalty}(k'+1,k)$

$W5_{\text{bifurcation-14}} = W5(i',k') + V(i'+2,i-1,k'+1,k-1) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{\text{dangle }k} + \Delta G^\circ_{gap} + \text{Penalty}(i'+2,i-1) + \text{Penalty}(k'+1,k-1)$

$W5_{\text{bifurcation-15}} = W5(i',k') + V(i'+2,i-1,k'+2,k) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{\text{dangle }k'+1} + \Delta G^\circ_{gap} + \text{Penalty}(i'+2,i-1) + \text{Penalty}(k'+2,k)$

$W5_{\text{bifurcation-16}} = W5(i',k') + V(i'+2,i-1,k'+2,k-1) + \Delta G^\circ_{\text{dangle }i'+1} + \Delta G^\circ_{\text{dangle }i} + \Delta G^\circ_{\text{dangle }k'+1} + \Delta G^\circ_{\text{dangle }k} + \text{Penalty}(i'+2,i-1) + \text{Penalty}(k'+2,k-1)$

$W5(0,k')$ is initialized to $\Delta G^o_{gap} \times k'$.

W3 is calculated in a similar manner to W5.

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